6.4 Connections: Absolute Value and Inequalities

What you should learn:

Goal 1 How to solve absolute value inequalities

Goal 2 How to model a real-life situation with an absolute value inequality

Why you should learn it:

You can use absolute value inequalities to model real-life situations about quantities within or outside of a number line interval, such as expenses being within 5% of budget.

Goal 1 Solving Absolute Value Inequalities

LESSON INVESTIGATION

■ Investigating Absolute Value Inequalities

Partner Activity Use a guess-and-check strategy to find values of x that satisfy each absolute value inequality.

a. $|x| < 2$ **b.** $|x + 2| \geq 1$ **c.** $|x - 3| \leq 2$

Sketch the solution set of each inequality on a number line. Then use a double inequality to describe the solution set.

In this investigation, you used a guess-and-check strategy to solve absolute value inequalities. The following properties give you another way to solve such inequalities.

Translating Absolute Value Inequalities

1. The inequality $|ax + b| < c$ is equivalent to

$$-c < ax + b < c.$$

2. The inequality $|ax + b| > c$ is equivalent to

$$ax + b < -c \quad \text{or} \quad ax + b > c.$$

Study Tip

The first property at the right is true when either < or ≤ is used. The second property is true when either > or ≥ is used.

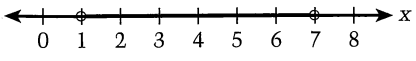
Example 1 Solving an Absolute Value Inequality

Solve $|x - 4| < 3$.

Solution

$$\begin{aligned} |x - 4| &< 3 && \textit{Rewrite original inequality.} \\ -3 < x - 4 &< 3 && \textit{Write equivalent compound inequality.} \\ 1 < \ x \ &< 7 && \textit{Add 4 to each expression.} \end{aligned}$$

The solution is *all real numbers that are greater than 1 and less than 7*. The graph is shown at the left. ■



Graphic Check

Example 2 Solving an Absolute Value Inequality

Solve $|x + 1| \geq 2$.

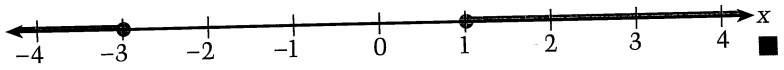
Solution This absolute value inequality is equivalent to the following compound inequality.

$$x + 1 \leq -2 \quad or \quad x + 1 \geq 2$$

You can solve this inequality using the procedure described in Lesson 6.3.

$x + 1 \leq -2$	***First inequality***	$x + 1 \geq 2$	***Second inequality***
$x \leq -3$	***Subtract 1.***	$x \geq 1$	***Subtract 1.***

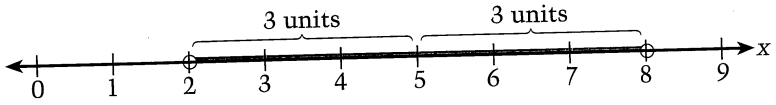
The solution is *all real numbers that are less than or equal to* -3 *or greater than or equal to* 1. The graph is shown below.



The next example shows how to write an absolute value inequality for a given graph.

Example 3 Writing an Absolute Value Inequality

Write an absolute value inequality to fit the graph shown below.



Solution To begin, write the compound inequality for the graph.

$$2 < x < 8$$

Then, to write the absolute value inequality, find the number that lies halfway between 2 and 8. This number is 5. (It lies 3 units from 2 and 3 units from 8.) Subtract 5 from each expression in the compound inequality.

$$2 - 5 < x - 5 < 8 - 5$$
$$-3 < x - 5 < 3$$

In this form, you can recognize the absolute value inequality to be

$$|x - 5| < 3.$$

For the inequality $|x - 5| < 3$, you can say that the range of solutions is from 3 less than 5 to 3 greater than 5. ■

Goal 2 Modeling a Real-Life Situation

```
10 PRINT "BUDGET EXP. ACTUAL EXP."
20    FOR N=1 TO 2
30    READ B,A: PRINT USING "$###, ###.## ";B,A;
40    IF ABS(B-A)<.05*B THEN PRINT "APPROVED" ELSE PRINT
      "QUERIED"
50 NEXT
60 END
70 DATA 124500, 130290.18, 36500, 34383.29
```

This computer program compares actual expenses with budgeted expenses. It flags any expense that differs from its budget by 5% or more.

Real Life Accounting

Example 4 *Budget Variance*

You are the manager of the accounting department of a large retail store and receive the following *budgeted* and *actual* expenses from the marketing department. How will the computer respond?

Item	Budgeted Expense, B	Actual Expense, A
a. Salaries	\$124,500.00	\$130,290.18
b. Travel	\$36,500.00	\$34,383.29

Solution

a. For salaries, $|B - A| = 5790.18$ and $0.05B = 6225.00$. The inequality $|B - A| < 0.05B$ is satisfied. The computer will print "APPROVED."

b. For travel, $|B - A| = 2116.71$ and $0.05B = 1825.00$. The inequality $|B - A| < 0.05B$ is *not* satisfied. The computer will print "QUERIED." ■

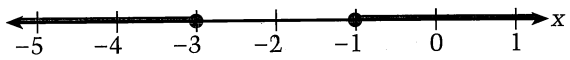
Communicating about ALGEBRA

SHARING IDEAS about the Lesson

Translate between Models Write an absolute value inequality for each graph.

A.

B.

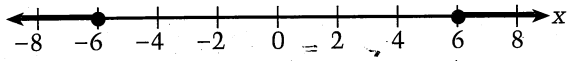


EXERCISES

Guided Practice

CRITICAL THINKING about the Lesson

1. **True or False?** $|x + 1| < 3$ means that $x + 1$ is between -3 and 3. Explain.
2. **True or False?** $|x| \geq 5$ means that x is less than -5 or greater than 5. Explain.
3. Solve $|x - 1| < 4$.
4. Solve $|x + 3| > 9$.
5. Match the inequality $|x| \geq 6$ with its graph.

a.


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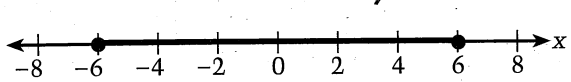
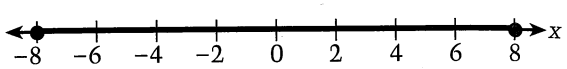


6. Match the inequality $|x| < 8$ with its graph.

a.


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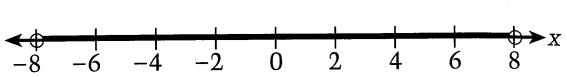


Independent Practice

In Exercises 7–12, match the absolute value inequality with its equivalent compound inequality.

7. $|x - 1| < 3$
8. $|x - \frac{1}{2}| \leq \frac{3}{2}$
9. $|x + 1| \leq 2$
10. $|x + 2| \geq 4$
11. $|x - \frac{5}{2}| > \frac{3}{2}$
12. $|x + 2| > 1$

a. $x < -3$ or $x > -1$
b. $-3 \leq x \leq 1$
c. $x \leq -6$ or $x \geq 2$
d. $-2 < x < 4$
e. $-1 \leq x \leq 2$
f. $x < 1$ or $x > 4$

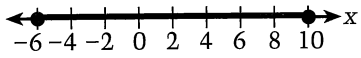
In Exercises 13–18, solve the inequality.

13. $|x - 2| < 5$
14. $|1 + x| \geq 4$
15. $|2x - 1| > 3$
16. $|4 + x| \leq 9$
17. $|4 - x| \geq 2$
18. $|-x + 1| < 1$

In Exercises 19–30, solve the inequality. Then sketch its graph.

19. $|x + 8| < 9$
20. $|9 + x| \leq 7$
21. $|4 - x| < 5$
22. $|x + 12| < 36$
23. $|x - 12| \geq 6$
24. $|5 - x| > 18$
25. $|x + 1| > 17$
26. $|x - 9| \geq 4$
27. $|3x - 6| \geq 0$
28. $|10 - 4x| < 2$
29. $|1 + 2x| \leq 9$
30. $|2x + 3| > 4$

In Exercises 31–36, write an absolute value inequality to fit the graph.

31.


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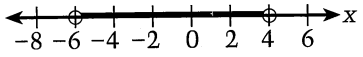

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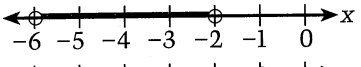

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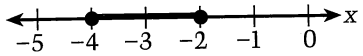

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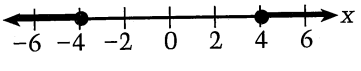

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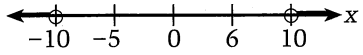


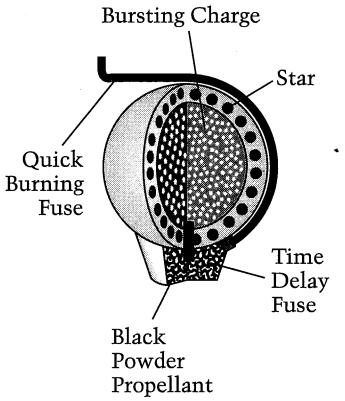
37. ***Game Show*** A contestant on a television show must guess within \$1000 of the actual price of a trip in order to win the trip. The actual price of the trip is \$14,000. Write an absolute value inequality that shows the range of possible guesses that will win the trip.

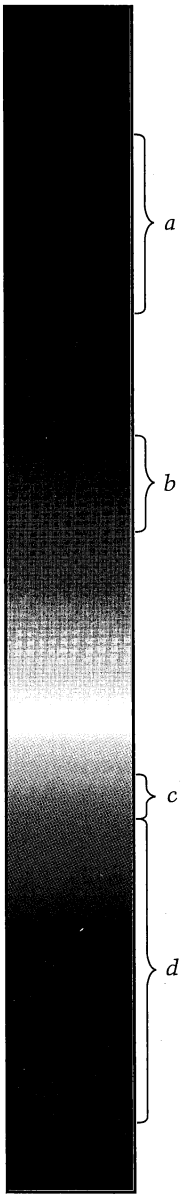
38. ***Car Mileage*** Your car averages 25 miles per gallon. The actual mileage for city driving varies from the average by 4 miles per gallon, depending on the traffic conditions. Write an absolute value inequality that shows the range for the mileage that your car gets in the city.

39. ***Test Scores*** Suppose the test scores in your class range from 60 to 100. Write an absolute value inequality describing test-score range.

40. ***400-Meter Run*** The members of your track team can run 400 meters between 58 seconds and 70 seconds. Write an absolute value inequality describing the times for the runners.

Fireworks **In Exercises 41–44, match the color of the star burst with one of the color ranges in the spectrum. (Use the wavelengths for various colors of the spectrum given on page 291.)**





A star-burst fireworks shell contains a quick fuse that ignites the propellant powder, expelling the shell from a launch tube. A time-delay fuse sets off a bursting charge after the shell is far above the ground. The color of the "stars" is determined by the chemical compound used.

41. A fireworks star is made of strontium. When it is burned, strontium emits light at wavelengths given by $|w - 643| < 38$. What color is it?

42. A fireworks star is made of a copper compound. When it is burned, the compound emits light at wavelengths given by $|w - 455| < 23$. What color is it?

43. A fireworks star is made of barium chloride. When it is burned, barium chloride emits light at wavelengths given by $|w - 519.5| < 12.5$. What color is it?

44. A fireworks star is made of a sodium compound. When it is burned, the compound emits light at wavelengths given by $|w - 600| < 5$. What color is it?